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Exercises for the Lecture Techniques in Artificial Intelligence

First Order Logic

1) One rich domain for the application of logical reasoning is e-commerce. Imagine that, rather than having humans in the purchasing and contracts departments of companies, we just let the machines do the negotation. Let's say we want to buy some pencils. We simply publish online the logical specification for the kind of pencils that we want to buy. The pencil vendors grab the specification and try to prove that they have pencils satisfying it; if so, they generate a bid for the pencils. This whole process is much easier for the buyer, who gets to state his requirements in general logical terms, rather than having to learn the part numbers of each kind of pencil from each vendor and ask for prices that way.

   Your job will be to write logical formulations of the following specifications for pencils. Your solutions should be of the form

$$\forall x_1, \ldots, x_n.\ \text{spec}(x_1, \ldots, x_n) \leftrightarrow \phi,$$

   where ϕ is a statement of the requirements on the items $x_1, \ldots, x_n$ under consideration:

   (a) a red or green pencil

   (b) two green pencils

   (c) a pencil being priced less than 0.25€

   (d) a pencil of highest quality (globally)

   (e) a pencil of highest quality (in your warehouse)

   (f) a pencil having the same color as a particular pencil P

   (g) a pencil having a unique color (globally)

   (h) a pencil having the same color as that one we (Pointy Pencil Corp, PPC) purchased the last time at your shop

   You should use First Order Logic in order to formulate the product specifications. You are allowed to use the following predicates, functions and constants, exclusively:

   - Predicates:
     - spec(x), spec($x_1, \ldots, x_n$): x or $(x_1, \ldots, x_n)$, respectlively, meets the specification
     - pencil(x): x is a pencil
     - red(x), green(x): x has the color red or green, respectively
     - inWarehouse(x): x is available in your warehouse
     - boughtBy(x, y): x was purchased by y
     - purchaseTime(x, t): x was purchased at time t
     - $x \leq y$: x is lower or equal than y
     - $x = y$: x and y are equal
   - Functions and constants:
     - price(x): Function returning the price of x (integer in cent)

– quality(x): Function returning the quality of x as an integer value (the larger the better)
– color(x): Function returning the color of an object x
– PPC: Constant identifying the Pointy Pencil Corp.

2) *Natural Language and First Order Formulas*

Assign the natural language sentences to the corresponding formulas encoding them. Here, $big(x)$ has the meaning "x is big", $\neg big(x)$ has the meaning "x is small" and $cat(x)$ has the meaning "x is a cat."

1. $\exists x.\ big(x) \wedge cat(x)$
2. $\exists x.\ big(x) \rightarrow cat(x)$
3. $\exists x.\ big(x) \vee cat(x)$
4. $\forall x.\ big(x) \wedge cat(x)$
5. $\forall x.\ big(x) \rightarrow cat(x)$
6. $\forall x.\ big(x) \vee cat(x)$
7. $\forall x.\ big(x) \vee \neg cat(x)$

(A) All cats are big.
(B) There exists a small thing or a cat.
(C) All big things are cats.
(D) All things are big cats.
(E) All cats are big.
(F) There exists a big cat.
(G) All small things are cats.
(H) There exists a big thing or a cat.

3) (a) Transform the following facts into first order logic.

(i) Every coyote chases some roadrunner.
(ii) Every roadrunner saying "beep-beep" is smart.
(iii) No coyote ever catches a smart roadrunner.
(iv) Every coyote chasing a roadrunner but not catching it, is frustrated.
(v) (Conclusion) If all roadrunners say "beep-beep", then all coyotes are frustrated.

You may use the predicates coyote/1, roadrunner/1, chases/2, catches/2, beep/1, smart/1 and frustrated/1.

(b) Show that the formulas actually entail the conclusions by conducting a proof by resolution

4) For each of the following logical formulas, decide whether or not it represents an appropriate translation of the corresponding natural language statement about rental prices and appartments. If not, point on the mistakes.

(a) "Every appartment in Munich is cheaper than some appartments in Starnberg."

$$\forall x.\ App(x) \wedge In(x, Munich) \rightarrow \exists y.\ App(y) \wedge In(x, Starnberg) \rightarrow Rent(x) < Rent(y)$$

(b) "There is exactly one appartment in Starnberg the rent of which is less than 500 €."

$$\exists x.\ App(x) \wedge In(x, Starnberg) \wedge (\forall y.\ App(y) \wedge In(y, Starnberg) \wedge Rent(y) < 500 \rightarrow x = y)$$

(c) "If some appartment is more expensive than any appartment in Munich, than it must be located in Starnberg."

$$\forall x.\ App(x) \wedge (\forall y.\ App(y) \wedge In(y, Munich) \wedge Rent(x) > Rent(y)) \rightarrow In(x, Starnberg)$$